Dynamic Correlation Functions of 1D Quantum Liquids

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in collaboration with

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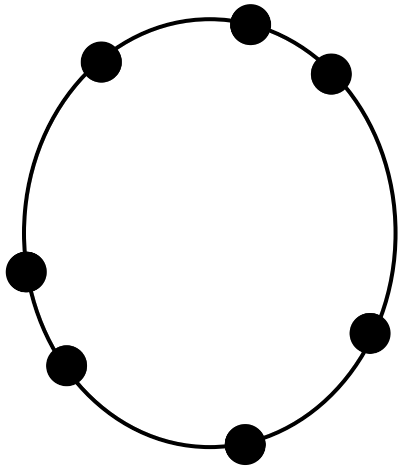
Michael Pustilnik, Georgia Tech

PRL **96**, 196405 (2006);
PRL **99**, 110405 (2007);
PRB (2007), PRA (2008).

Models



N –interacting quantum particles on a ring

$$\hat{H} = -\frac{1}{2m}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2} + \sum_{i<j} V(x_i - x_j)$$

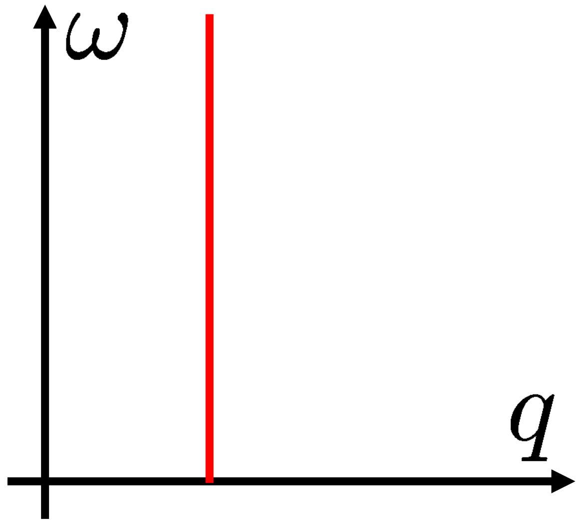
- ✓ Thermodynamic limit
- ✓ Bosons or spinless fermions
- ✓ Translationally invariant
- ✓ Bosons with $V(x_i - x_j) = c\delta(x_i - x_j)$ Lieb-Liniger, integrable
- ✓ (1+1)D complex field theory, ``critical''

Observables

Density

$$\hat{\rho}(x) = \sum_{i=1}^{N} \delta(x - x_i)$$

$$\hat{\rho}(x,t) = e^{-i\hat{H}t}\rho(x,0)e^{i\hat{H}t}$$

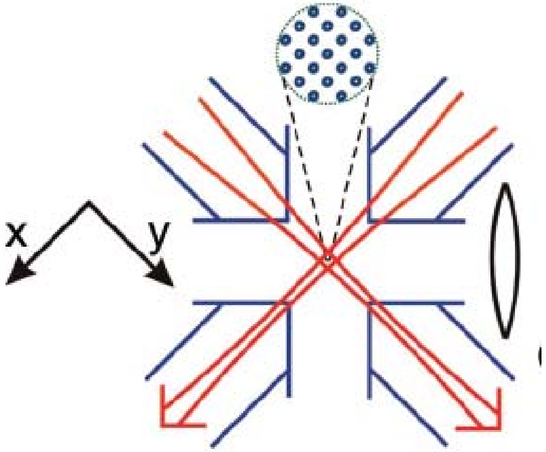


Dynamic Structure Factor (DSF) at T=0

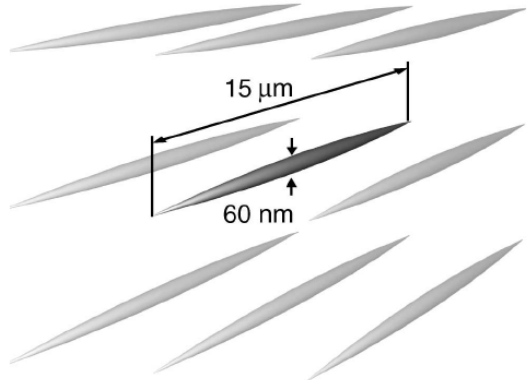
$$S(x,t) = \langle 0|\rho(x,t)\rho(0,0)|0\rangle$$

$$S(q,\omega) = \int dx\, dt\, e^{i(qx-\omega t)} S(x,t) = \sum_{n_q} |\langle n_q|\hat{\rho}|0\rangle|^2 \delta(\omega - \epsilon_{n_q})$$

Cold Atoms in Optical Lattices

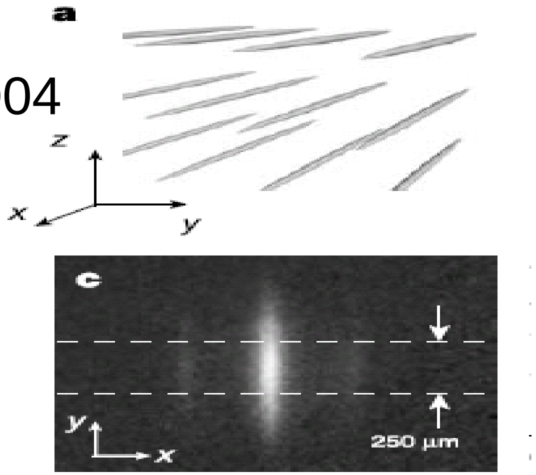


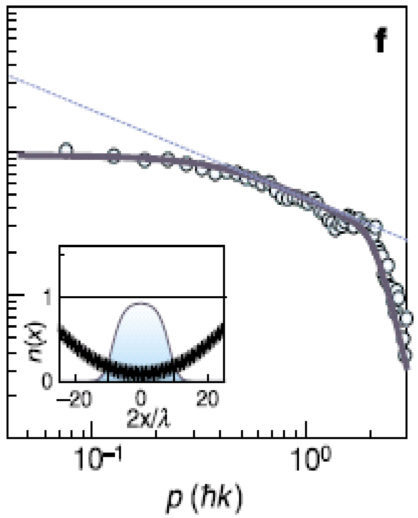
T. Kinoshita, et al 2004



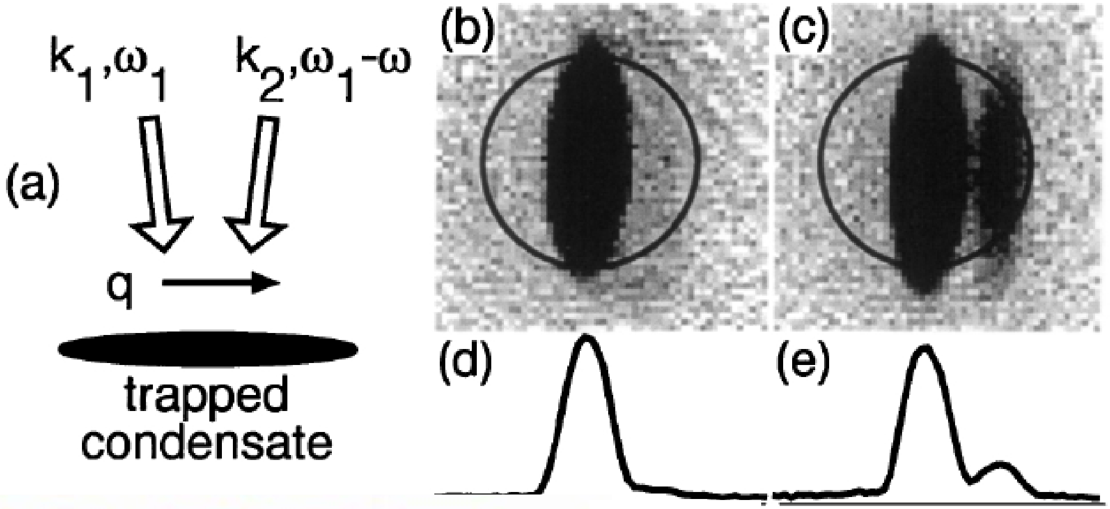
H. Moritz, et al 2003

I. Bloch, et al 2004

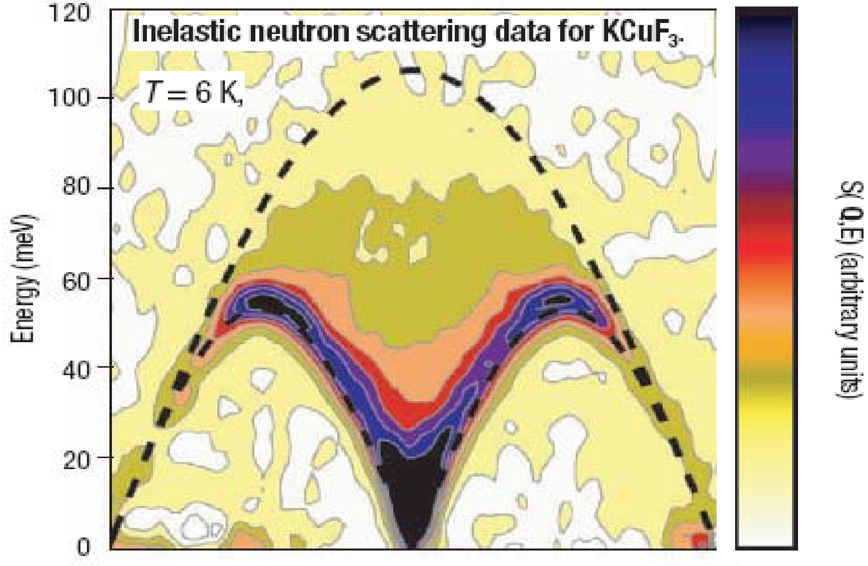


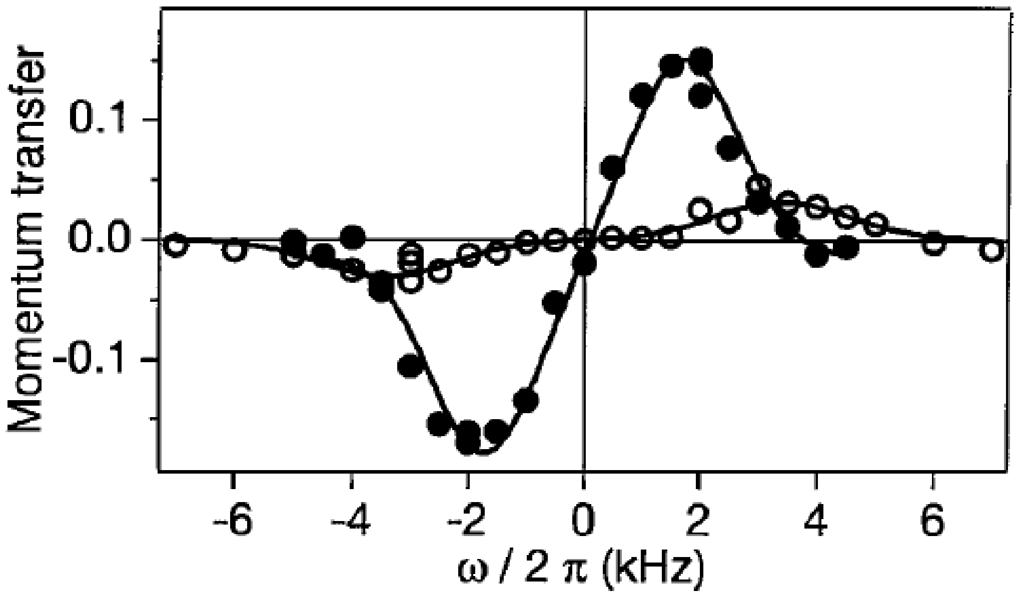


Bragg Scattering



W. Ketterle, et al
2000-…



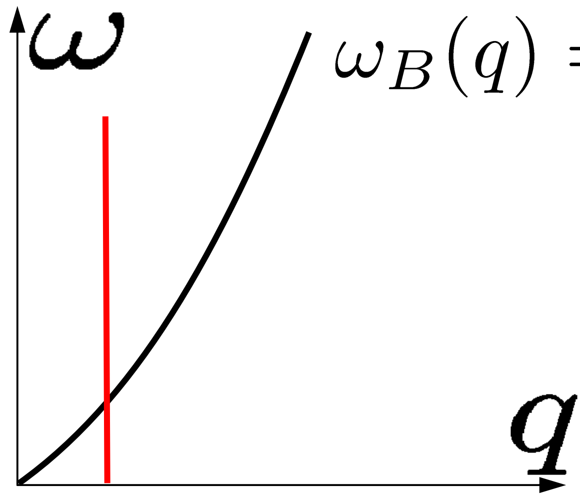


BELLA LAKE[1,2*†], D. ALAN TENNANT[2,3‡],
CHRIS D. FROST[3] AND STEPHEN E. NAGLER[1]

nature materials | VOL 4 | APRIL 2005

3D Condensates

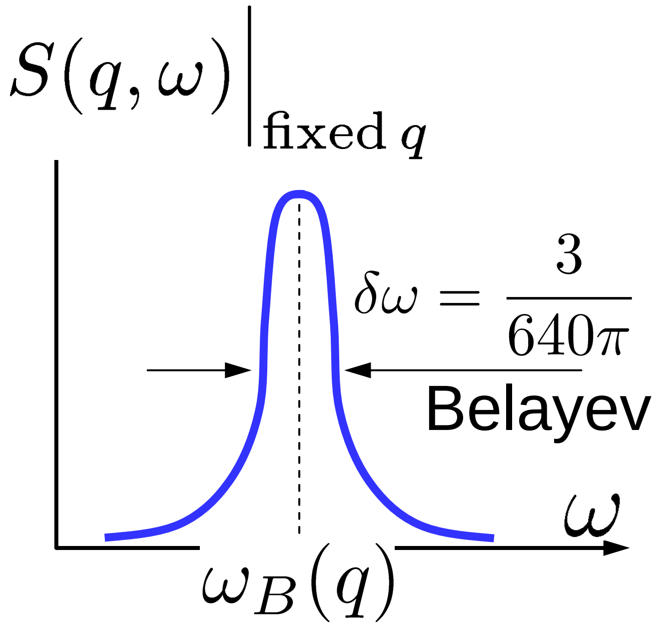
$$T < T_c$$



$$\omega_B(q) = v_s q \sqrt{1 + (q/2mv_s)^2}$$

Bogoliubov mode

✓ What about 1D?



$$\delta\omega = \frac{3}{640\pi}\,\frac{q^5}{\hbar^3 nm} \ll \omega_B$$

Belayev 1958

PHYSICAL REVIEW VOLUME 130, NUMBER 4 15 MAY 1963

Exact Analysis of an Interacting Bose Gas. I. The General Solution and the Ground State

ELLIOTT H. LIEB AND WERNER LINIGER
Thomas J. Watson Research Center, International Business Machines Corporation, Yorktown Heights, New York
(Received 7 January 1963)

- ✓ N bosons with delta-functional interactions on a 1D ring

$$H = -\frac{1}{2m}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2} + c\sum_{i<j}\delta(x_i - x_j)$$

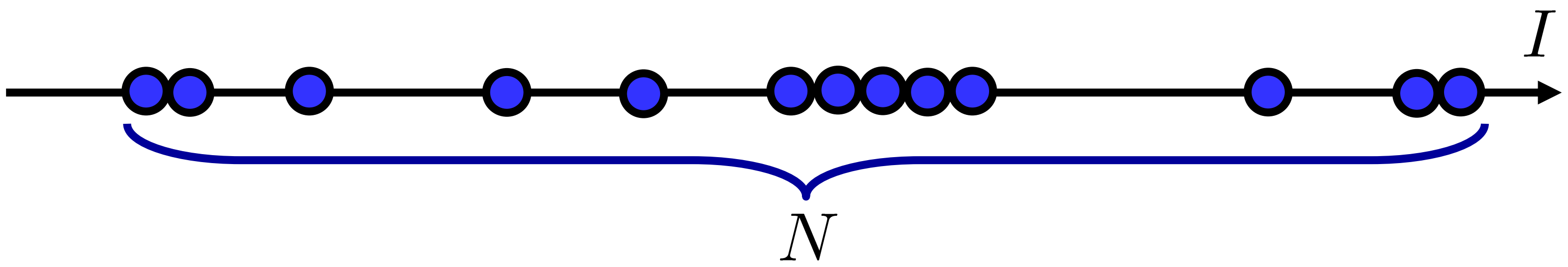
- ✓ Two characteristic momenta: mc and n =N/L

- ✓ Dimensionless coupling constant: $\gamma = \frac{mc}{n}$

Bethe Ansatz

$$\psi(x_1, x_2, \ldots, x_N) = \sum_P a(P)\, e^{\,i \sum_{j=1}^{N} x_j \lambda_j}$$

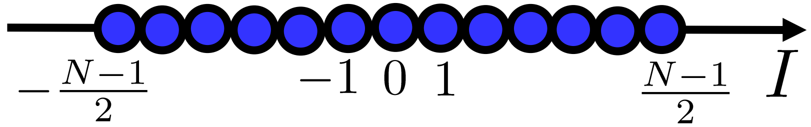
$$\lambda_j + \frac{1}{L} \sum_k 2 \arctan \frac{\lambda_j - \lambda_k}{m\, c} = \frac{2\pi}{L} I_j$$ ← integers



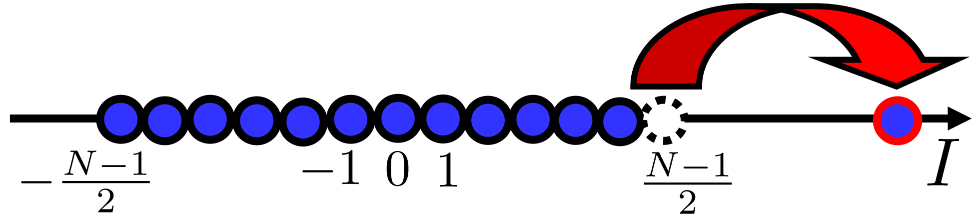
$$E = \frac{1}{2m} \sum_j \lambda_j^2 \qquad\qquad q = \sum_j \lambda_j = \frac{2\pi}{L} \sum_j I_j$$

Lieb's Modes

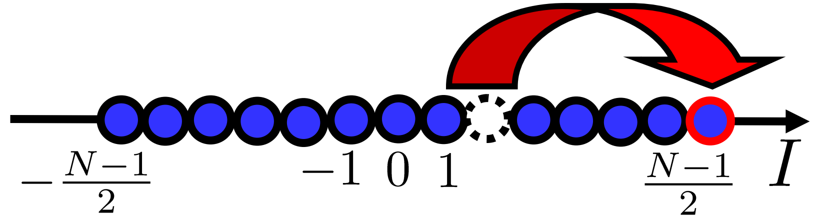
Ground state:

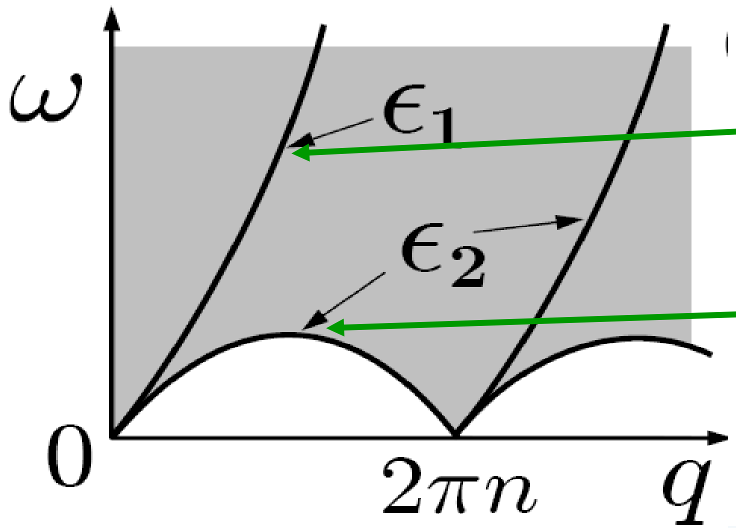


Lieb's I mode
"particles":



Lieb's II mode
"holes":

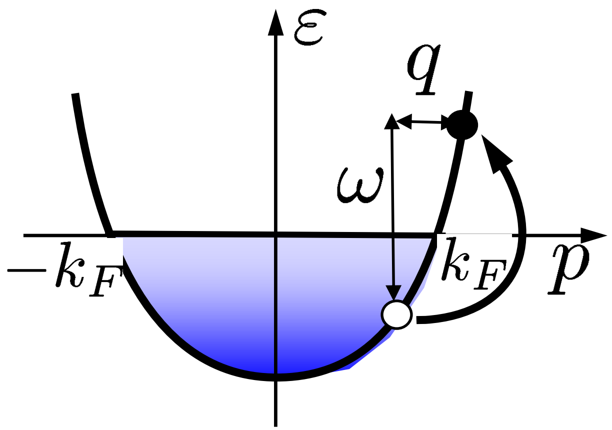




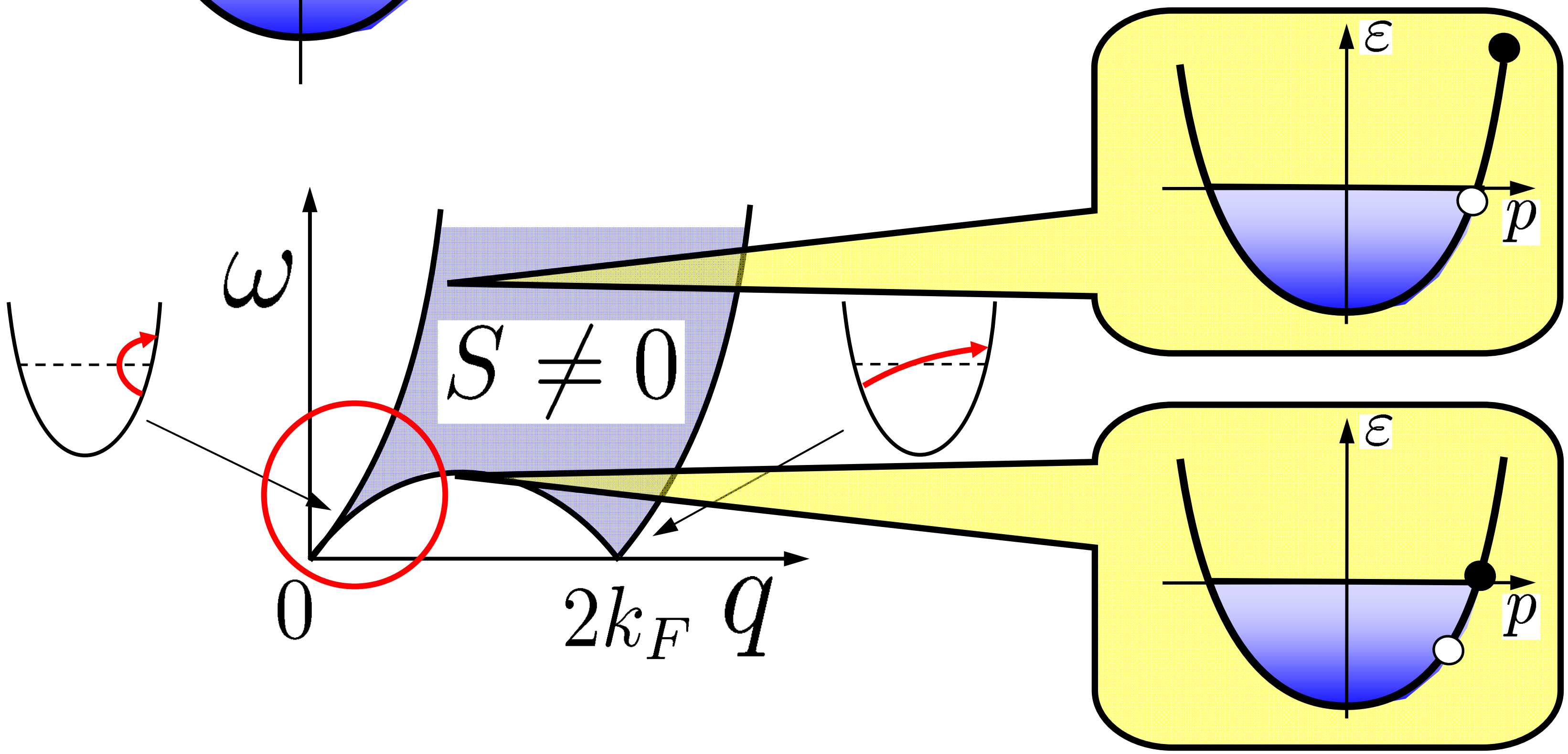
Bogoliubov for $\gamma \to 0$

Lower bound of the spectral continuum

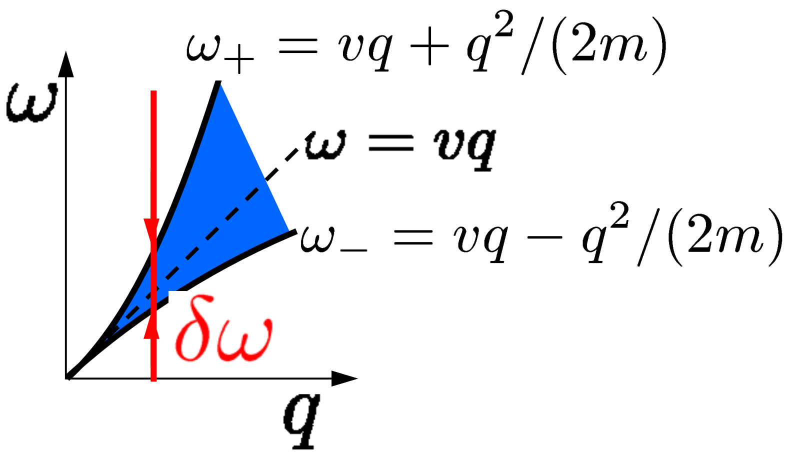
Strongly Interacting Bosons = Free Fermions

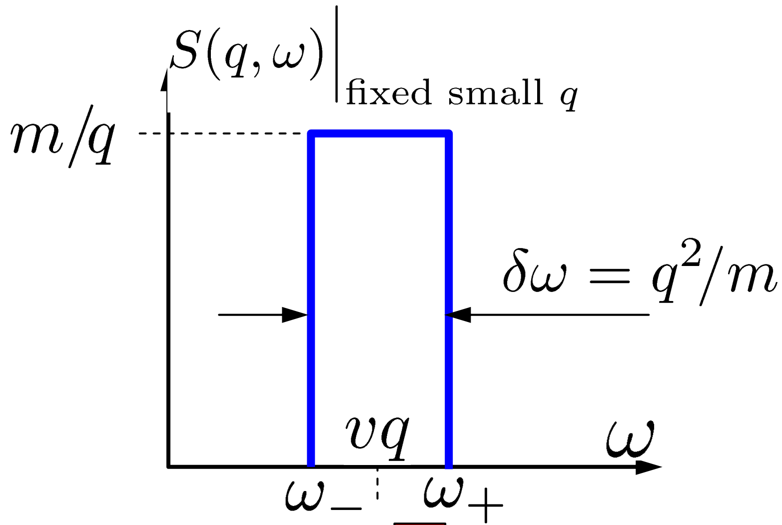


$$S(q,\omega) = \left\langle \rho(q,\omega)\rho(-q,-\omega) \right\rangle$$

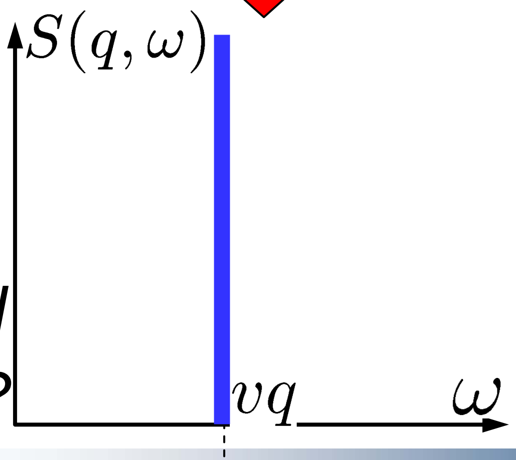


Structure Factor (free fermions)





Linear dispersion

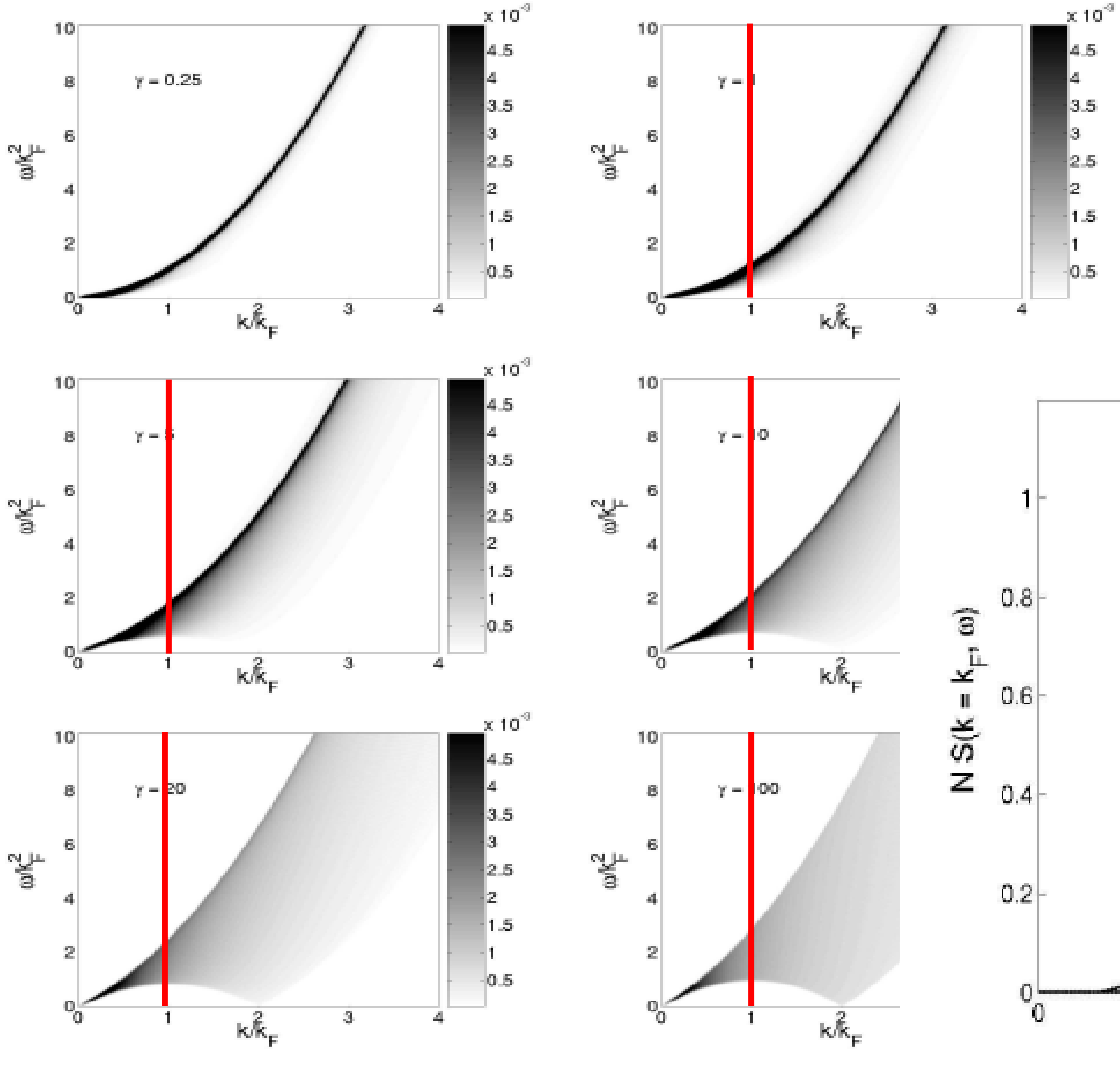


$$S(q,\omega) = q\delta(\omega - vq)$$

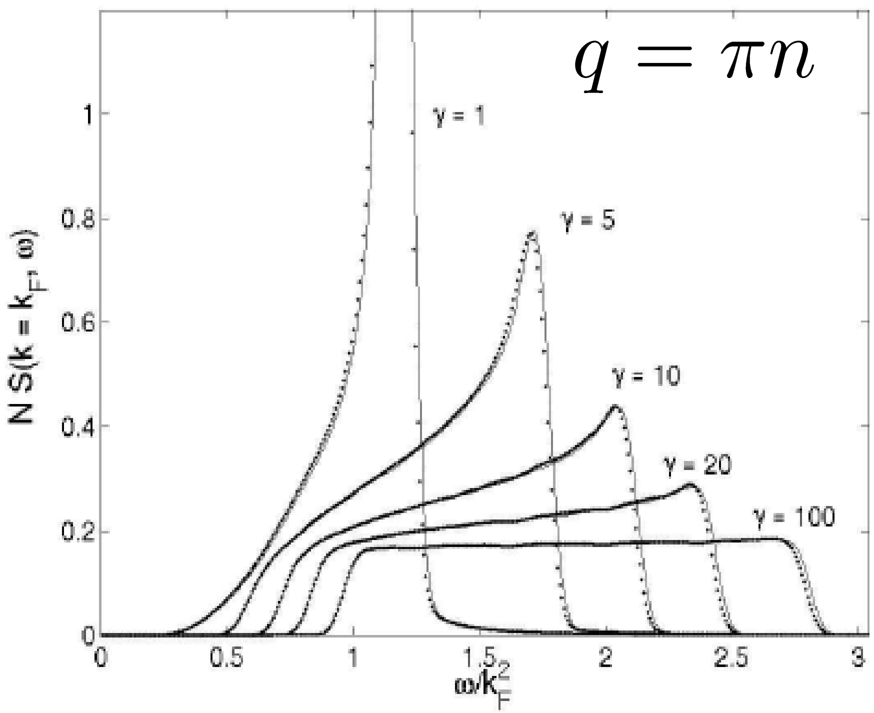
✓ **Exact result within the Luttinger approximation.**

How does the dispersion curvature and interactions affect the structure factor ?

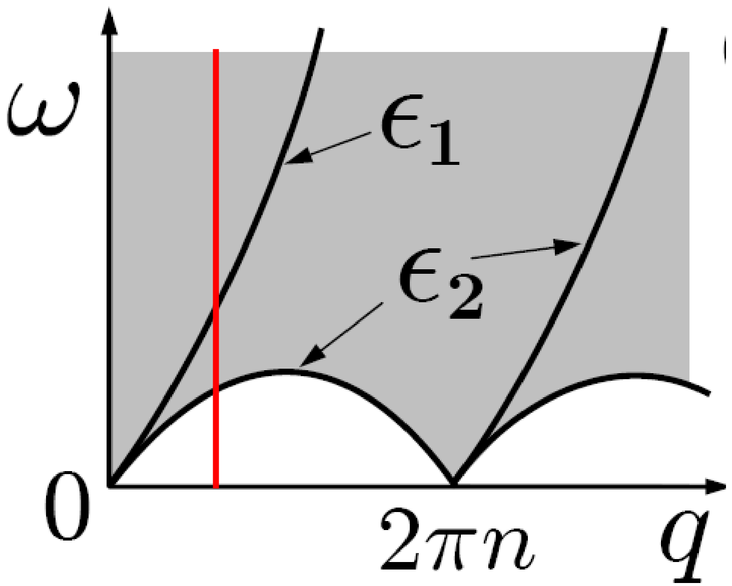
Algebraic BA exact numerics

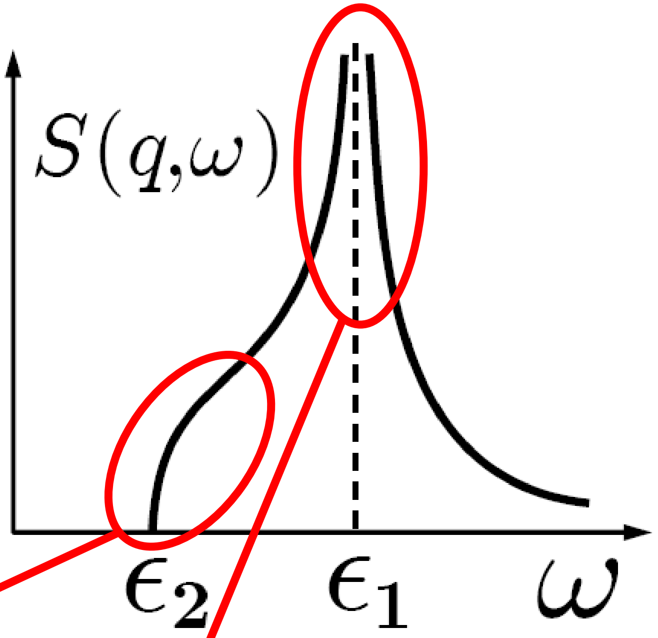


J-S. Caux, P. Calabrese, 2006
N. Slavnov, 1989



DSF singularities at Lieb's modes





$$S(q,\omega) \sim \frac{m}{q} \left[\frac{\omega - \epsilon_2}{\delta\epsilon} \right]^{\mu_2(q)} \theta(\omega - \epsilon_2)$$

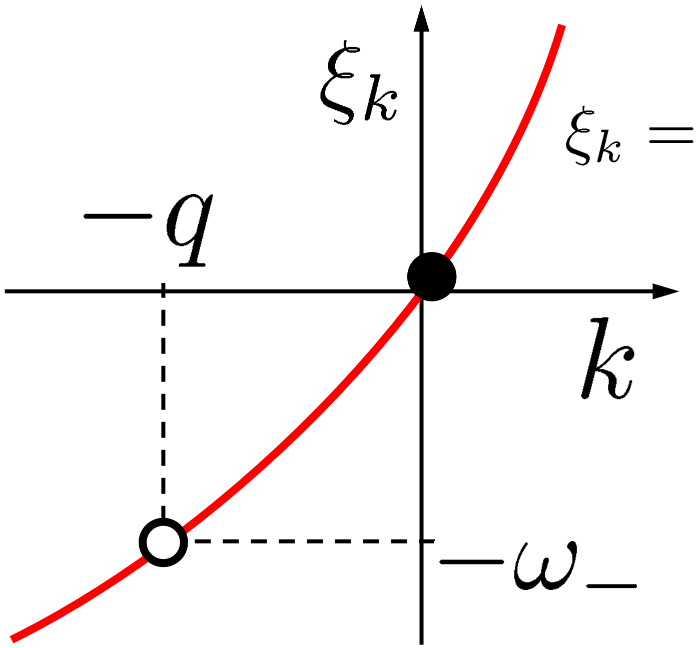
$$\delta\epsilon = \epsilon_1 - \epsilon_2$$

$$S(q,\omega) \sim \frac{m}{q} \left| \frac{\delta\epsilon}{\omega - \epsilon_1} \right|^{\mu_1(q)} \left[\theta(\epsilon_1 - \omega) + \nu_1 \theta(\omega - \epsilon_1) \right]$$

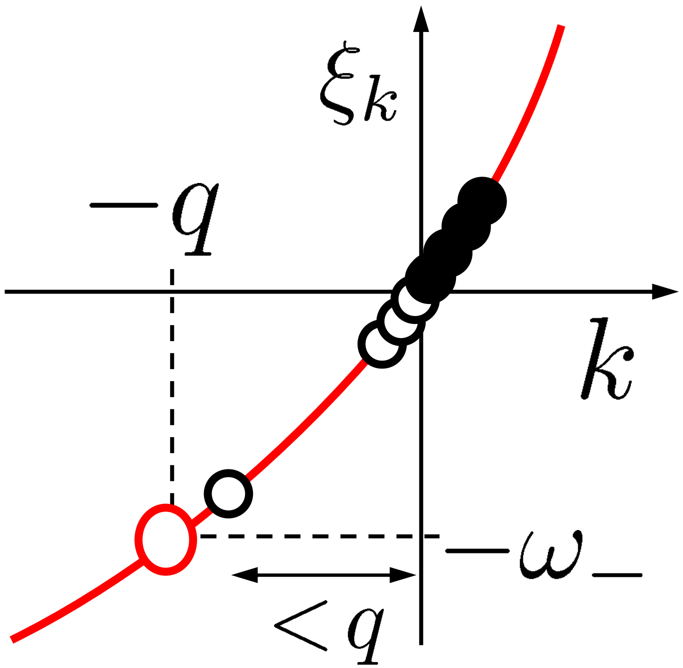
$$\mu_1 < 1$$

Effective model $\omega_- \lesssim \omega$

multipair states with momentum q



$$\xi_k = vk + \frac{k^2}{2m}$$



single pair

energy $\to \omega_-$

states, contributing to the leading logarithm corrections in each order of the perturbation theory.

Mahan 67, Nozieres-DeDominicis 69

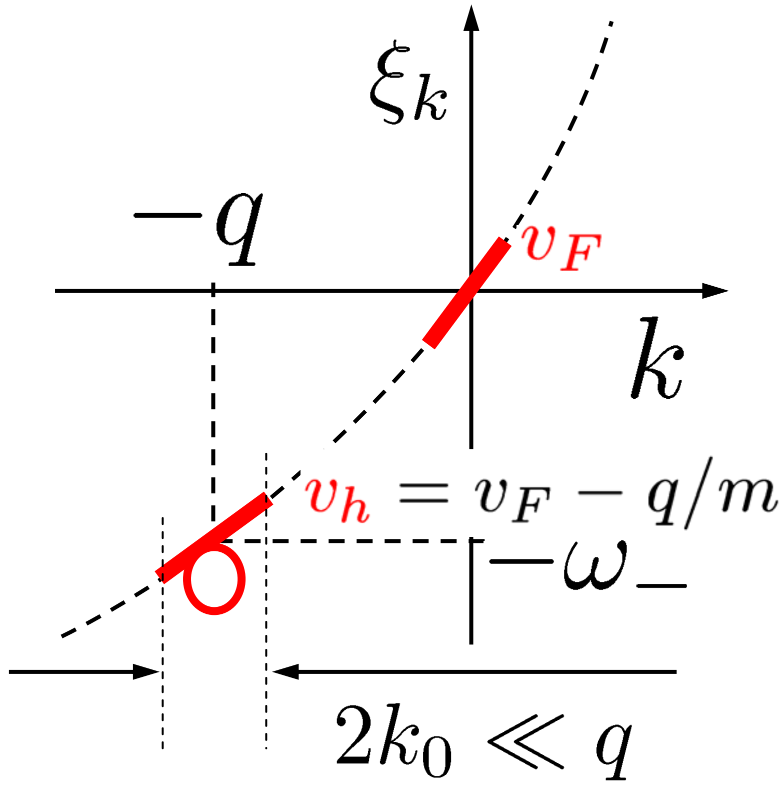
Effective model $\omega_- \lesssim \omega$

multipair states with momentum q

Single deep hole +
low-energy excitations

Fermi-edge singularity
problem.

Power-law edge
singularities.



the idea: project all other states out;
linearize remaining spectum.

Mahan 67, Nozieres-DeDominicis 69

Why Power-Law ?

Band of low energy excitations:

$$H_0 = \frac{v}{2\pi} \int dx \left[\frac{1}{K} (\partial_x \phi)^2 + K (\partial_x \theta)^2 \right]$$

Haldane 1981

Deep hole creation operator
(instantaneous shift of density and current):

$$\hat{D}(x,t) = e^{i[\delta_\theta \phi(x,t) + \delta_\phi \theta(x,t)]}$$

Dynamic structure factor

power-law of $x - v_d t$

$$S(q,\omega) = \int dx\, dt\, e^{i(qx-\omega t)} \left\langle \hat{D}(x,t) \hat{D}^\dagger(0,0) \right\rangle_{H_0}$$

Exactly solvable models

$\delta_{\phi,\theta}(q)$ can be determined by comparing finite size spectrum of the effective model and Bethe Ansatz spectrum with fixed total momentum q

Pereira, White, Affleck, 2008, 2009
Cheianov, Pustilnik, 2008
Imambekov, Glazman, 2008
Khodas, 2009

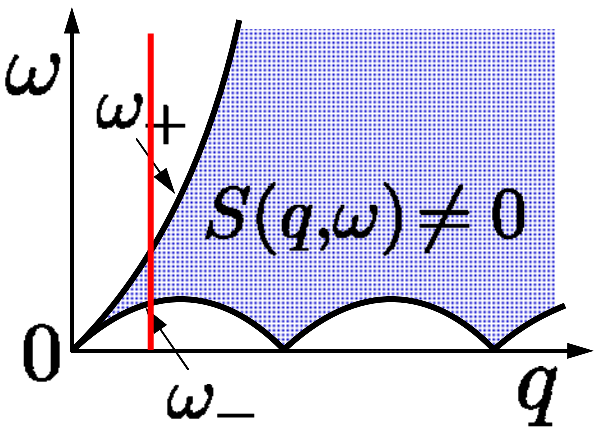
Calogero-Sutherland model

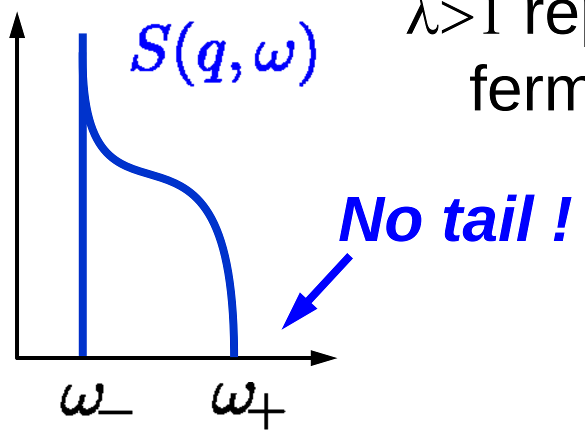
$$H = -\sum_i \frac{1}{2m}\frac{\partial^2}{\partial x_i^2} + \sum_{i<j} V(x_i - x_j) \qquad V(x) = \frac{\lambda(\lambda-1)\pi^2/m}{\sin^2(\pi x)}$$

Haldane 1994, Ha 1995 $\lambda = \frac{k}{n}$ S =k +n dim

integral

λ>1 repulsive fermions





$S(q,\omega)$: power-law singularities at $\omega \to \omega_\pm$

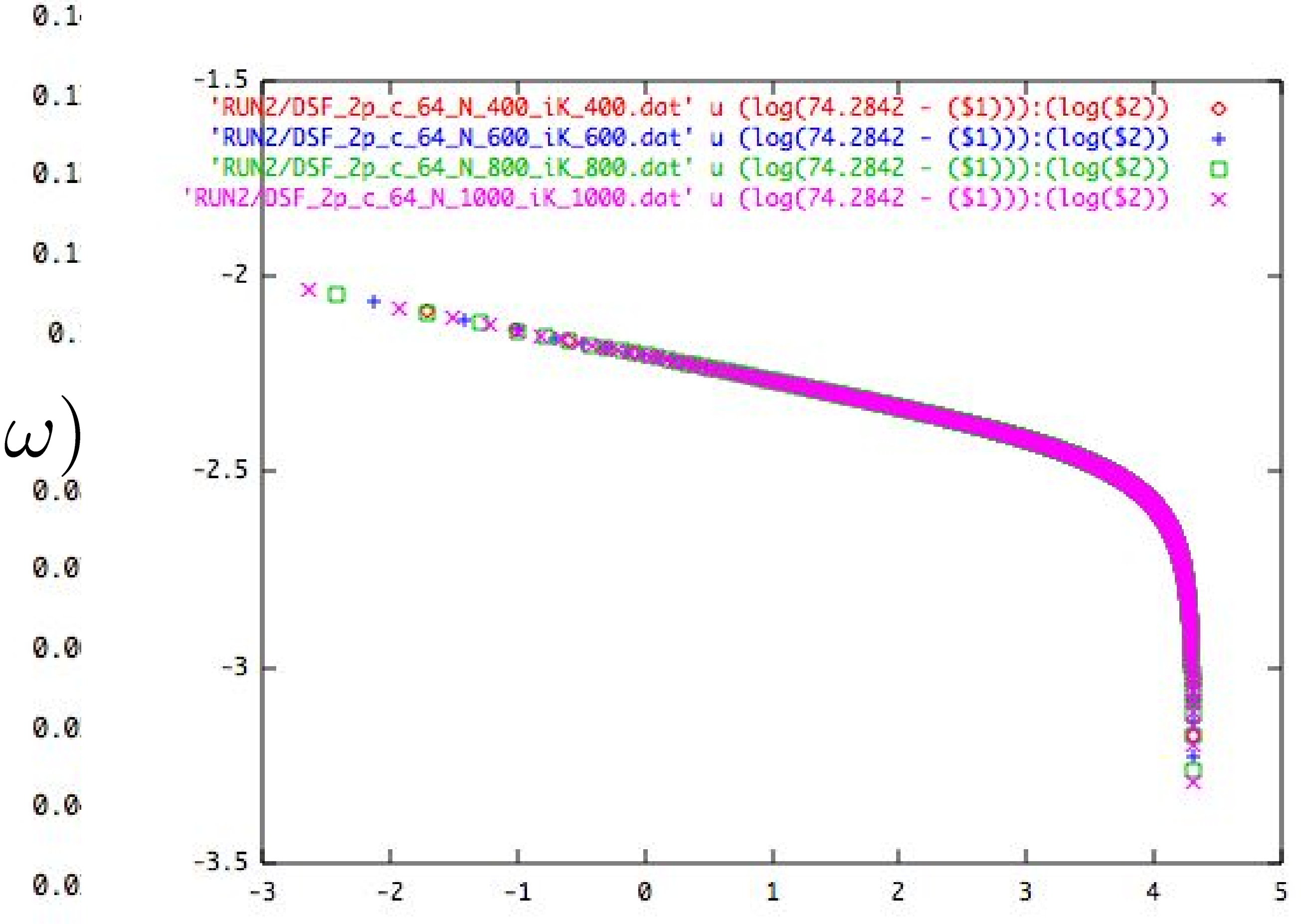
$$S \propto (\omega - \omega_-)^{1/\lambda - 1} \qquad S \propto (\omega_+ - \omega)^{\lambda - 1}$$

Pustilnik, 2006

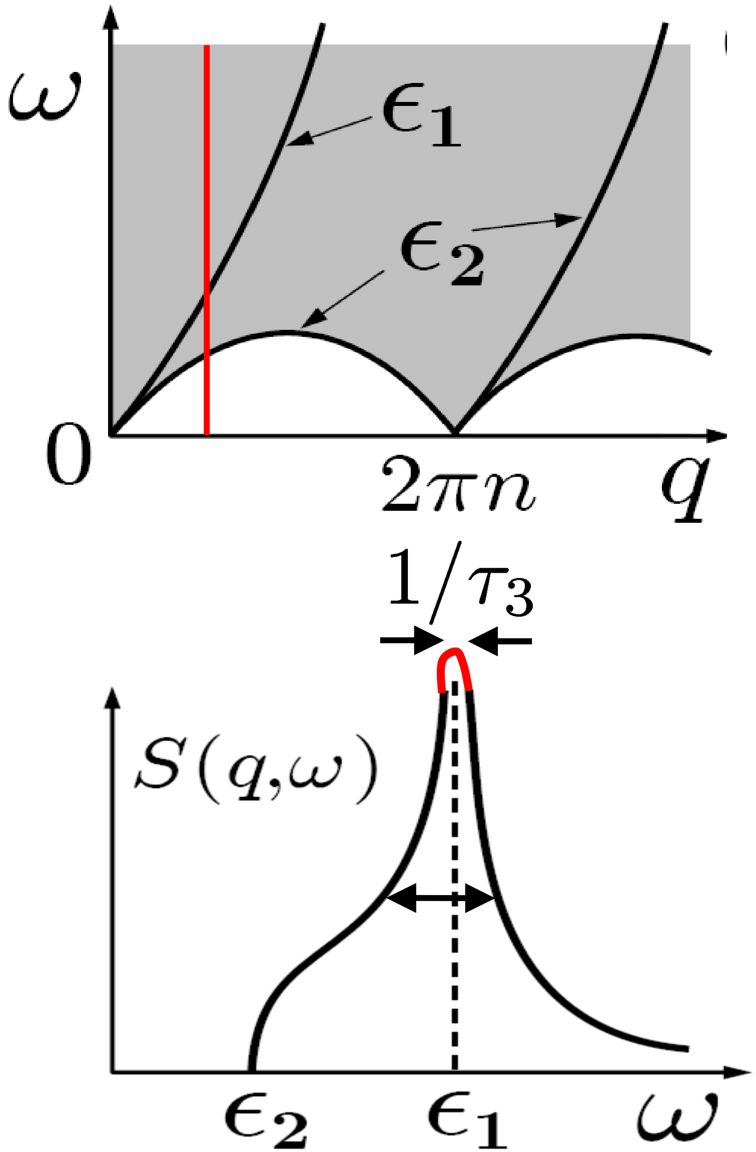
Numerics (preliminary)

Courtesy of J-S. Caux

$S(2\pi n, \omega)$

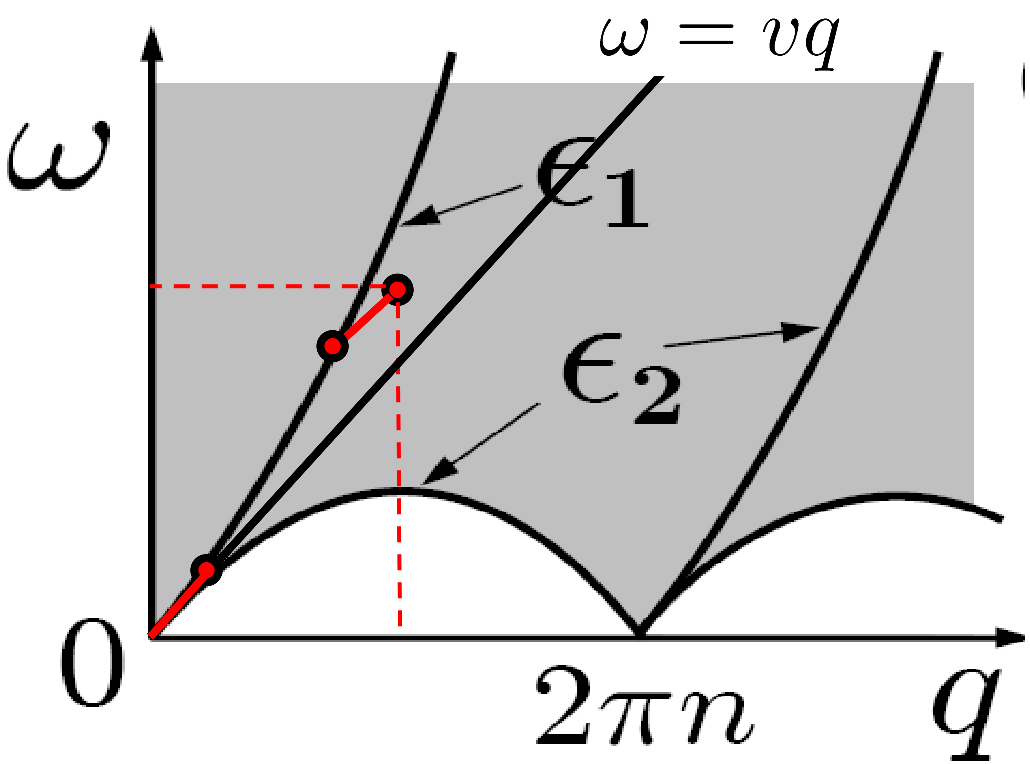


Singularities at Lieb's modes



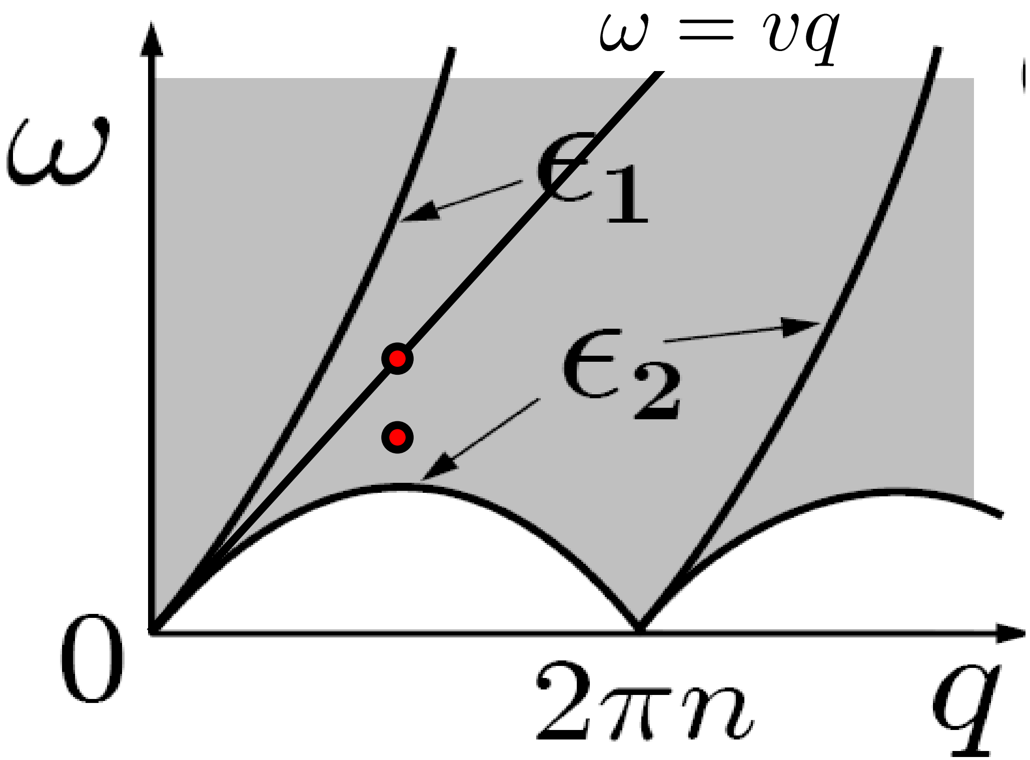
- ✓ For integrable models: <u>positions</u> and <u>exponents</u> are known from TBA
- ✓ Scaling functions ???
- ✓ Singularities are NOT smeared by temperature
- ✓ Non-integrable models: three-body scattering smears singularity in the bulk, but NOT at the edge

Weakly Interacting Bosons



Phonon modes shake-up → power-law singularity

Weakly Interacting Bosons



For $\omega = vq$ **infinite number of quasiparticles is excited.**

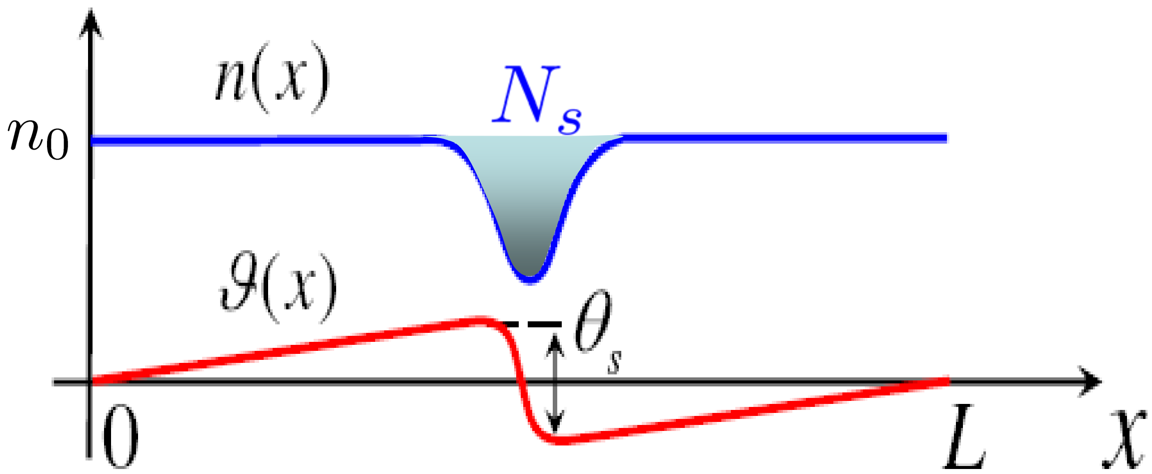
What are the excitations below $\omega = vq$ **line ?**

Dark Solitons

$$i\partial_t \psi = -\frac{1}{2m}\nabla^2\psi + c|\psi^2|\psi - \mu\psi$$

Gross-Pitaevskii equation

$$\psi(x - Vt) = \sqrt{n(x - Vt)}\, e^{i\vartheta(x-Vt)}$$

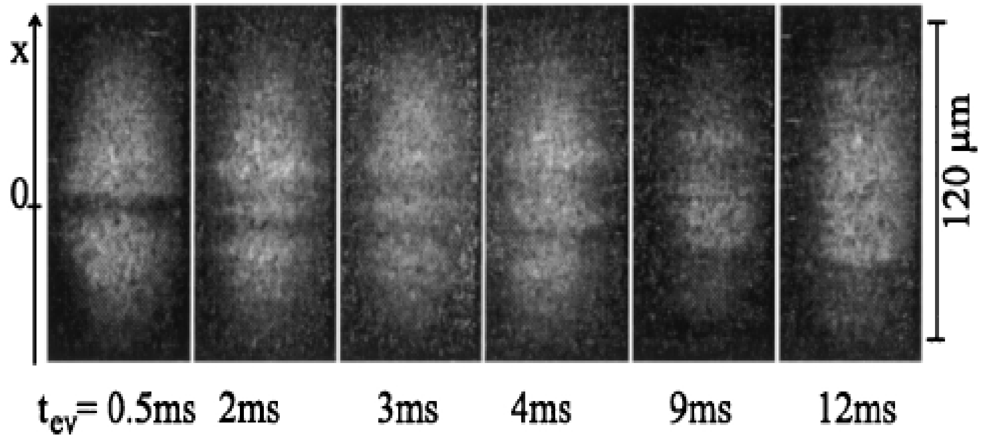


$$N_s = \frac{2}{\sqrt{\gamma}}\sqrt{1 - \frac{V}{v_B}} \gg 1$$

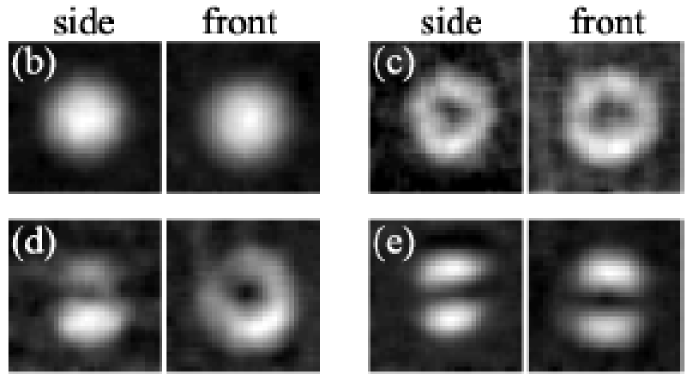
$$\theta_s = 2\arccos(V/v_B)$$

$$p_s = n_0\left(\theta_s - \sin\theta_s\right)\;; \qquad \varepsilon_s = \frac{4}{3} n_0 v_B \sin^3(\theta_s/2)$$

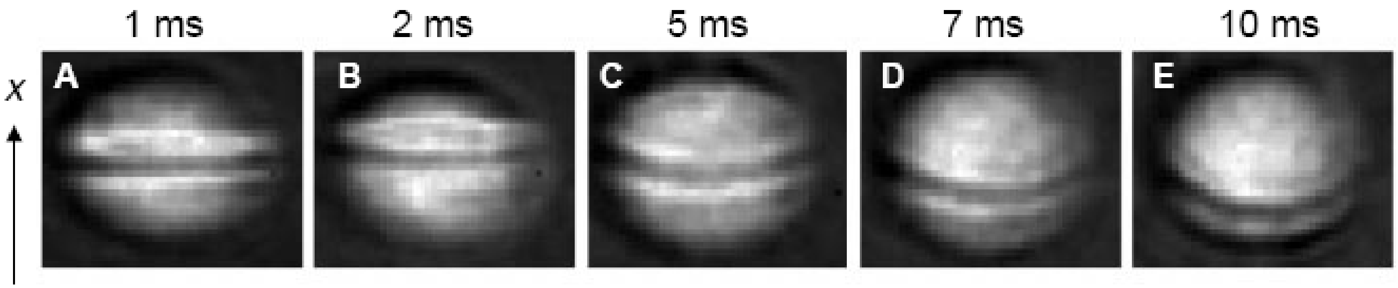
Observations of Dark Solitons



Sengstock, et al. 1999



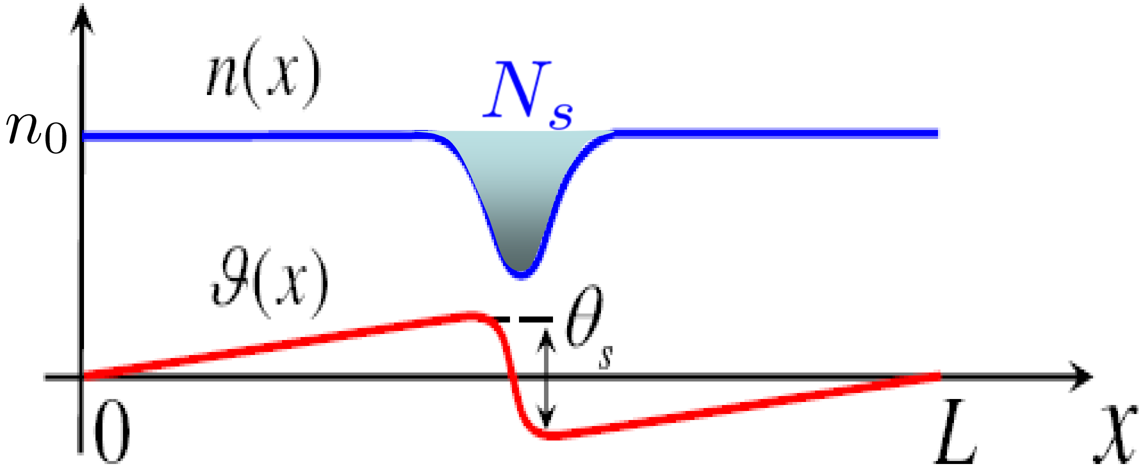
Cornell, et al. 2001

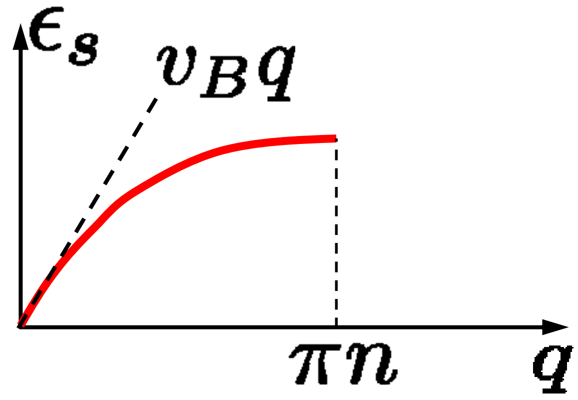


Phillips, et al. 2000

Dark Solitons as Lieb II excitations

$$p_s = n_0\,(\theta_s - \sin\theta_s)\ ; \qquad \varepsilon_s = \frac{4}{3} n_0 v_B \sin^3(\theta_s/2)$$





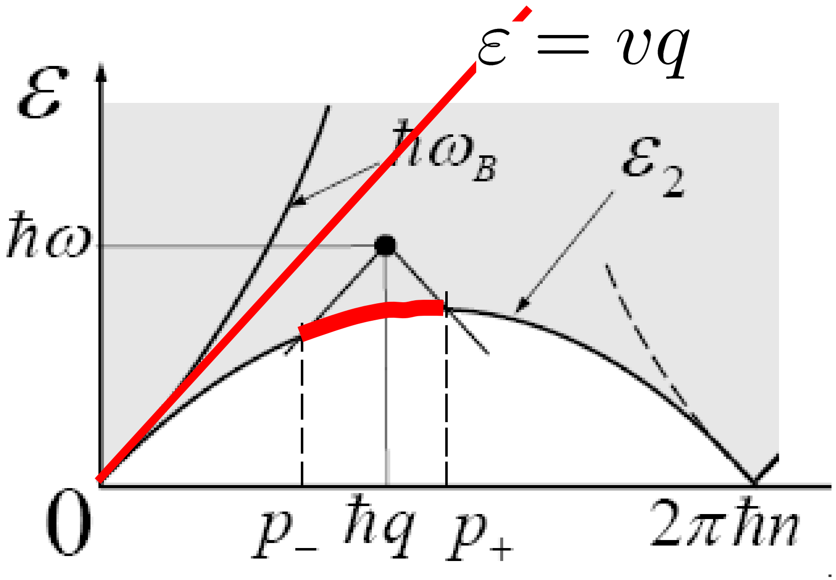
$$\epsilon_s(q) \overset{\gamma \ll 1}{\rightarrow} \epsilon_2(q)$$

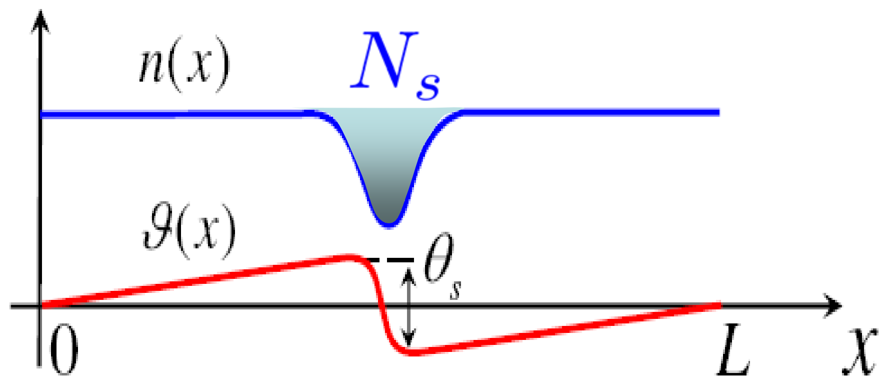
Dark soliton

Lieb II mode

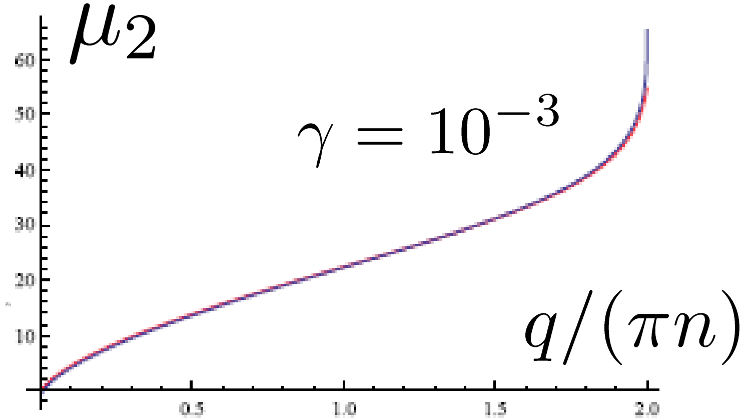
P.P. Kulish, S.V. Manakov and L.D. Faddeev 1976

Photo-Solitonic Effect





$$S \sim (\delta\varepsilon)^{\mu_2}$$



Probability to excite a soliton is suppressed by orthogonality

Wake up!

- ✓ **Power-law singularities at Lieb modes, where exponents are functions of q**

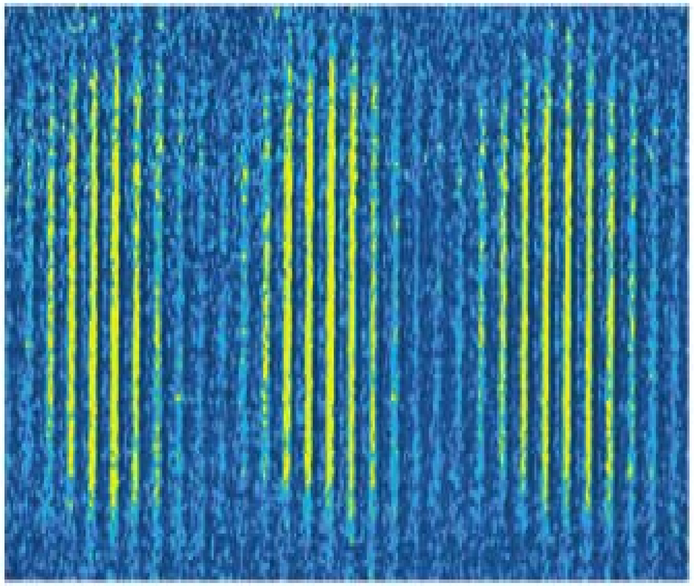
- ✓ **Single energetic particle + low-energy excitations**

- ✓ **Single particle = quasiparticle or soliton**

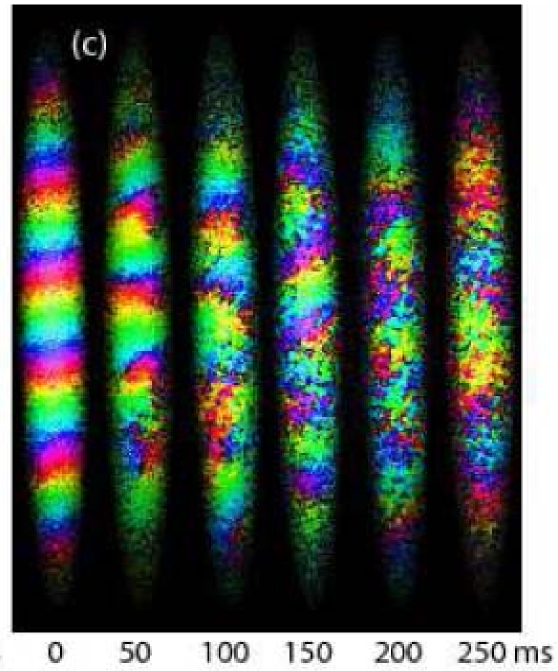
- ✓ **Photo-Solitonic Effect**

1D Spinor Condensates

Hyperfine states, e.g. F=1. Ferromagnetic ground state

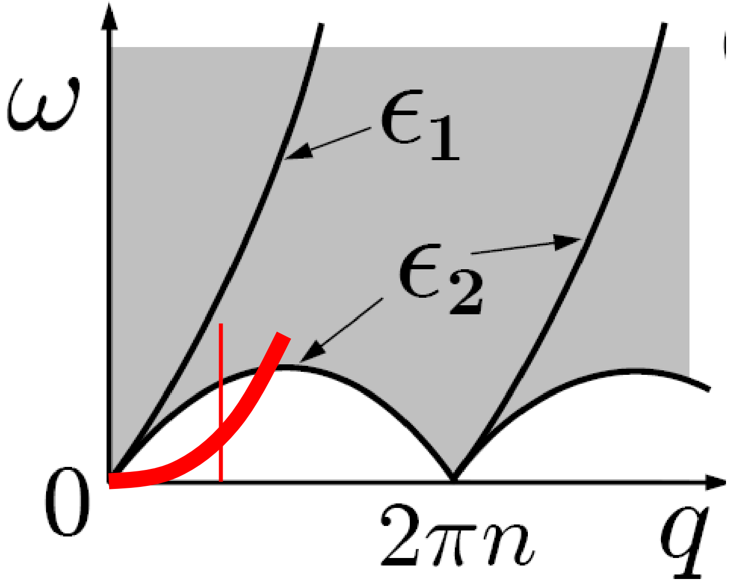


J.M. Higbie, et al 2008



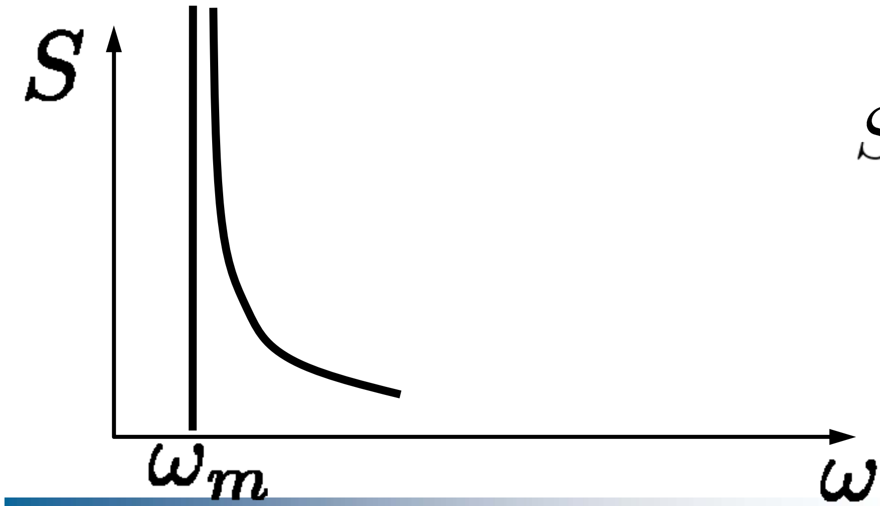
M. Vingalatore, et al 2008

1D Spinor Condensates



Ferromagnetic magnon

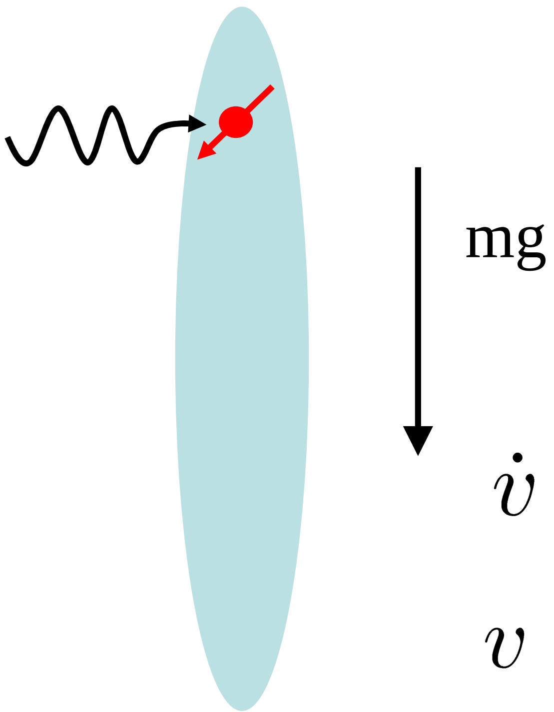
$$\omega_m = \frac{q^2}{2m^*}$$

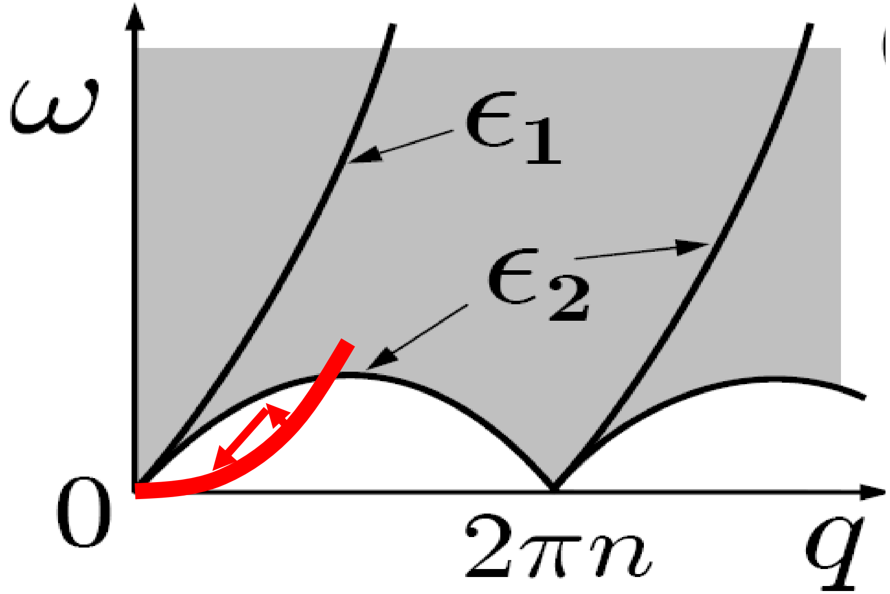


$$S(q,\omega) \sim \frac{1}{(\omega - \omega_m(q))^{\mu_m(q)}}$$

Gravitational Fall in Spinor Condensate

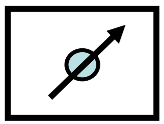
M. Kohl





$$\dot{v} = g - \Gamma v$$

$$v = g/\Gamma \sim T^{-4}\left(\frac{v_g}{v_B}\right)^3$$



detector

$$\frac{\hbar}{m v_g} = \frac{v_g^2}{g}; \qquad v_g = \left(\frac{4.1}{A}\right)^{1/3} \frac{cm}{s}$$

Atomic number